$\rho^0 - \omega$ mixing in $\mathrm{U}(3)_L \times \mathrm{U}(3)_R$ chiral theory of mesons

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Abstract. In this paper, $\mathrm{U}(3)_L \times \mathrm{U}(3)_R$ chiral theory of mesons is extended to the leading order in quark mass expansion in order to evaluate the $\rho^0 - \omega$ mixing. It is shown that the use of path integral method to integrate out the quark fields naturally leads to the $\rho^0 - \omega$ mixing vertices, and this mixing is entirely from the quark loop in this theory. The off-shell behaviour of the mixing amplitude is analyzed. The on-shell mixing amplitude is obtained from the decay $\omega \to \pi^+\pi^-$. Furthermore, the constraints on the light quark mass parameters are extracted from the $\rho^0 - \omega$ mixing and the mesons masses, and the mass splitting of $K^*(892)$-mesons is predicted.

1 Introduction

In the limit of vanishing light quark masses, the lagrangian of quantum chromodynamics (QCD) possesses the exact chiral $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ symmetry. It has been known that this symmetry is spontaneously broken to $\mathrm{SU}(3)_V$ with the appearances of eight Goldstone pseudoscalar particles (π, K, η) which dominate low energy dynamics of the strong interaction. Chiral perturbation theory (ChPT), which is expanded in powers of derivatives of the mesons fields, is rigorous and phenomenologically successful in describing the physics of the pseudoscalar mesons at very low energies [1]. On the other hand, chiral symmetry is explicitly broken due to the small current quark masses, which leads to the nonzero masses of the pseudoscalar mesons. In addition, the inequality of the light quark masses, especially, $m_u \neq m_d$, does break the isospin symmetry or charge symmetry. This breaking of isospin symmetry induces various hadron mixings such as $\pi^0 - \eta$, $\rho^0 - \omega$, and $\Lambda - \Sigma^0$ mixings etc [2]. In this paper, we will focus on the $\rho^0 - \omega$ mixing, which is considered as the important source of charge symmetry breaking in nuclear physics.

This investigation of $\rho^0 - \omega$ mixing has been an active subject [2–14], and the mixing amplitude for on-mass-shell vector mesons has been observed directly in the measurement of the pion form-factor in the time-like region from the process $e^+e^- \to \pi^+\pi^-$ [16]. For roughly twenty years, $\rho^0 - \omega$ mixing amplitude was assumed constant or momentum independent, even if ρ and ω mesons have the space-like momenta, far from the on-shell point. Several years

ago, this assumption was firstly questioned by Goldman, Henderson, and Thomas, and the mixing amplitude was found to be significantly momentum dependent within a simple quark loop model [8]. Subsequently, various authors have argued such q^2 dependence of the $\rho^0 - \omega$ mixing amplitude (where q^2 denotes the four-momentum square of the vector mesons) by using various theoretical approaches [9,10]. In particular, the authors of [10] has pointed out that $\rho^0 - \omega$ mixing amplitude must vanish at $q^2 = 0$ within a broad class of models.

The purpose of the present paper is to study the $\rho^0 - \omega$ mixing amplitude in the framework of $\mathrm{U}(3)_L \times \mathrm{U}(3)_R$ chiral theory of mesons [17,18]. This theory could be regarded as a realization of current algebra, chiral symmetry, and vector meson dominance (VMD). The meson fields including pseudoscalar, vector, and axial-vector mesons are introduced into the present theory as the bound state of quark fields. The effective lagrangian is obtained by using the path integral method to integrate out the quark fields, and the kinetic terms of the mesons are generated from quark loop naturally (see [17,18] for details). The present theory has been investigated extensively, and the theoretical results agree with the experimental data well [19–23]. Particularly, in [22], starting from the $\mathrm{U}(3)_L \times \mathrm{U}(3)_R$ chiral fields theory of mesons, and by using path integration method to integrate out the vector and axial-vector resonances, the authors have derived the chiral coupling constants of ChPT (L_1, L_2, L_3, L_9 and L_{10}). The results are in good agreement with the experimental values of the

L_i at $\mu = m_\rho$ in ChPT. Therefore, the QCD constraints discussed in [24] are met by this theory.

It has been known that $\rho^0 - \omega$ mixing amplitudes receive the contributions from two sources: isospin symmetry breaking due to u-d quark mass difference and electromagnetic interactions. As mentioned in [17], VMD [25, 26] in the meson physics is natural consequence of the present theory instead of an input. Therefore, the dynamics of the electromagnetic interactions of mesons has been well introduced and established. Thus, the calculation of $\rho^0 - \omega$ mixing amplitude from the transition $\rho \to \gamma \to \omega$ is straightforward. In [17,18], the light quark masses are set to be massless. When the current quark mass terms, which explicitly break the chiral symmetry, are included in the present theory, the use of path integral method to integrate out the quark fields will naturally reduce the $\rho^0 - \omega$ mixing terms in the mesons effective lagrangian (see below). Therefore, at the leading order in quark mass expansion and $O(\alpha_{\rm EM})$, the amplitude of $\rho^0 - \omega$ mixing can be evaluated systematically within U(3)$_L$×U(3)$_R$ chiral theory of mesons. As will be shown below, the $\rho^0 - \omega$ mixing receives the contributions entirely from the quark loop in the framework of the present theory.

In [7], in order to calculate the $\rho^0 - \omega$ mixing, the author extended the chiral couplings of the low-lying vector resonances in chiral perturbation theory [15] to a lagrangian that contains two vector fields, and made some assumptions to pin down the coupling constants of the lagrangian related to this mixing.

The contents of the paper are organized as follows. In Sec. 2, we present the basic notations of the U(3)$_L$×U(3)$_R$ chiral theory of mesons, and extend the theory to the leading order in quark mass expansion in order to derive the $\rho^0 - \omega$ mixing due to isospin symmetry breaking. In Sec. 3, the off-shell and on-shell $\rho^0 - \omega$ mixing amplitude are studied, and the constraints on the light quark mass parameters are extracted. In Sec. 4, we give a summary of the results.

2 U(3)$_L$×U(3)$_R$ chiral theory of mesons

The basic lagrangian of U(3)$_L$×U(3)$_R$ chiral theory of mesons is (hereafter we use the notation of [17,18])

$$\mathcal{L} = \bar{\psi}(x)(i\gamma \cdot \partial + \gamma \cdot v + e_0 Q\gamma \cdot A$$
$$+ \gamma \cdot a\gamma_5 - M - mu(x))\psi(x)$$
$$+ \frac{1}{2}m_1^2(\rho_i^\mu \rho_{\mu i} + \omega^\mu \omega_\mu + a_i^\mu a_{\mu i} + f^\mu f_\mu)$$
$$+ \frac{1}{2}m_2^2(K_\mu^{*a} K^{*a\mu} + K_1^\mu K_{1\mu})$$
$$+ \frac{1}{4}m_3^2(\phi_\mu \phi^\mu + f_s^\mu f_{s\mu}) + \mathcal{L}_{\rm EM} \tag{1}$$

with

$$u(x) = \exp[i\gamma_5(\tau_i\pi_i + \lambda_a K^a + \eta + \eta')],$$

$$a_\mu = \tau_i a_\mu^i + \lambda_a K_{1\mu}^a + (\frac{2}{3} + \frac{1}{\sqrt{3}}\lambda_8)f_\mu$$
$$+ (\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8)f_{s\mu},$$
$$v_\mu = \tau_i \rho_\mu^i + \lambda_a K_\mu^{*a} + (\frac{2}{3} + \frac{1}{\sqrt{3}}\lambda_8)\omega_\mu$$
$$+ (\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8)\phi_\mu, \tag{2}$$

where i=1, 2, 3 and a=4, 5, 6, 7. The ψ in (1) is u, d, s quark fields. M=diag(m_u, m_d, m_s) is the quark mass matrix, which represents the explicit chiral symmetry breaking in this theory. m is a parameter related to the quark condensate. $\mathcal{L}_{\rm EM}$ is the lagrangian of the electromagnetic interaction. A_μ is the photon field, and Q is the electric charge operator of the quark fileds. Note that there are no kinetic terms in (1) for the meson fields including pseudoscalar (π, K, η), vector (v_μ), and axial-vector (a_μ) because they are composed fields of quark fields instead of the fundamental fields. The kinetic terms for these fields will be generated from quark loops.

Following [17], the effective lagrangian of mesons (indicated by $\mathcal{M}$) are obtained by performing path integrations over the quark fields,

$$\exp\{i \int d^4x \mathcal{L}^{\mathcal{M}}\} = \int [d\psi][d\bar{\psi}]\exp\{i \int d^4x \mathcal{L}\}. \tag{3}$$

Using the dimensional regularization, and in the chiral limit, i.e. light quark masses are zero, the effective lagrangian $\mathcal{L}_{RE}$ (normal parity part) and $\mathcal{L}_{IM}$ (abnormal parity part) has been evaluated in [17,18]. Here we give the first two terms of $\mathcal{L}_{RE}$ in order to present the notations explicitly.

$$\mathcal{L}_{RE} = \frac{N_C}{(4\pi)^2}m^2\frac{D}{4}\Gamma(2 - \frac{D}{2})Tr D_\mu U D^\mu U^\dagger$$
$$- \frac{1}{3}\frac{N_c}{(4\pi)^2}\frac{D}{4}\Gamma(2 - \frac{D}{2})Tr(v_{\mu\nu}v^{\mu\nu} + a_{\mu\nu}a^{\mu\nu}) + ..., \tag{4}$$

where $U = \exp i(\tau_i\pi_i + \lambda_a K^a + \eta + \eta')$, and

$$D_\mu U = \partial_\mu U - i[v_\mu, U] + i\{a_\mu, U\},$$
$$D_\mu U^\dagger = \partial_\mu U^\dagger - i[v_\mu, U^\dagger] - i\{a_\mu, U^\dagger\},$$
$$v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu] - i[a_\mu, a_\nu],$$
$$a_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu - i[a_\mu, v_\nu] - i[v_\mu, a_\nu].$$

It is obvious that quark loop integral gives rise to the divergences in the effective lagrangian (4). In [17,18], in order to build a physical effective meson theory, a universal coupling constant g has been introduced

$$\frac{F^2}{16} = \frac{N_c}{(4\pi)^2}m^2\frac{D}{2}\Gamma(2 - \frac{D}{2}), \tag{5}$$

$$g^2 = \frac{8}{3}\frac{N_c}{(4\pi)^2}\frac{D}{4}\Gamma(2 - \frac{D}{2}) = \frac{1}{6}\frac{F^2}{m^2}. \tag{6}$$

and

$$\frac{F^2}{f_\pi^2}(1 - \frac{2c}{g}) = 1, \tag{7}$$

$$c = \frac{f_\pi^2}{2gm_\rho^2}, \qquad (8)$$

where f_π is the decay constant of pions.

It is straightforward to extend the effective lagrangian of the present theory to the leading order in quark mass expansion. Starting from (1), in which the quark mass terms are included, and using the similar procedure presented in [17], we can get the effective lagrangian beyond the chiral limit. The nonzero quark masses will yield other terms in addition to the effective lagrangian in the chiral limit. At the leading order in quark mass expansion, the masses of the pseudoscalar mesons will be no longer zero,

$$m_{\pi^+}^2 = m_{\pi^0}^2 = -\frac{2}{f_\pi^2}(m_u + m_d)\langle 0|\bar{\psi}\psi|0\rangle, \qquad (9)$$

$$m_{K^+}^2 = -\frac{2}{f_\pi^2}(m_u + m_s)\langle 0|\bar{\psi}\psi|0\rangle, \qquad (10)$$

$$m_{K^0}^2 = -\frac{2}{f_\pi^2}(m_d + m_s)\langle 0|\bar{\psi}\psi|0\rangle, \qquad (11)$$

$$m_\eta^2 = -\frac{2}{3f_\pi^2}(m_u + m_d + 4m_s)\langle 0|\bar{\psi}\psi|0\rangle, \qquad (12)$$

where $\langle 0|\bar{\psi}\psi|0\rangle$ is the quark condensate of the light flavors [17,21].

For the vector mesons, at the leading order in quark mass expansion, there are no the explicit contributions of their masses. Consequently, there is no such $\rho^0 - \omega$ mixing which is independent of the momentum (see below). However, an additional kinetic term of the vector mesons will appear in the effective lagrangian. Therefore, to the order of m_q, all the kinetic terms of the vector mesons induced from (1) and (3) are as follows,

$$\mathcal{L}_{kin}^V = -\frac{1}{8}g^2 Tr v_{\mu\nu} v^{\mu\nu} + \frac{2N_c}{3(4\pi)^2 m} Tr M v_{\mu\nu} v^{\mu\nu}$$
$$+\text{higher order terms.} \qquad (13)$$

The second term in (13) contains the $\rho^0 - \omega$ mixing due to isospin symmetry breaking, which can be easily derived

$$\mathcal{L}_{\rho\omega} = \frac{1}{4\pi^2}\frac{m_u - m_d}{m}\rho_{\mu\nu}^0 \omega^{\mu\nu}. \qquad (14)$$

However, the vector meson fields in (13) is not physical fields, therefore we should make the kinetic terms of the vector mesons fields in the standard form by redefining these fields. For ρ-mesons, from (13), we have

$$\mathcal{L}_{kin}^\rho = -\frac{1}{4}g^2(1 - \frac{1}{2\pi^2 g^2}\frac{m_u + m_d}{m})\rho_{\mu\nu}^i \rho^{i\mu\nu}. \qquad (15)$$

The physical ρ-mesons field should be defined as

$$\rho_\mu \longrightarrow \frac{1}{g\sqrt{1 - \frac{1}{2\pi^2 g^2}\frac{m_u + m_d}{m}}}\rho_\mu.$$

Then, the physical mass of ρ mesons is

$$m_\rho^2 = \frac{m_1^2}{g^2(1 - \frac{1}{2\pi^2 g^2}\frac{m_u + m_d}{m})} = \frac{m_V^2}{1 - \frac{1}{2\pi^2 g^2}\frac{m_u + m_d}{m}}. \qquad (16)$$

Expanding (16) to the first order of m_q, we obtain

$$m_\rho = m_V(1 + \frac{1}{4\pi^2 g^2}\frac{m_u + m_d}{m}), \qquad (17)$$

here $m_V = \frac{m_1}{g}$ is the vector meson masses in the chiral limit. Likewise, for ω, K^*, and ϕ mesons, we can get

$$m_\omega = m_V(1 + \frac{1}{4\pi^2 g^2}\frac{m_u + m_d}{m}), \qquad (18)$$

$$m_{K^{*\pm}} = m_V(1 + \frac{1}{4\pi^2 g^2}\frac{m_u + m_s}{m}), \qquad (19)$$

$$m_{K^{*0}} = m_V(1 + \frac{1}{4\pi^2 g^2}\frac{m_d + m_s}{m}), \qquad (20)$$

$$m_\phi = m_V(1 + \frac{1}{2\pi^2 g^2}\frac{m_s}{m}). \qquad (21)$$

In the derivation of the above equations, we have set $m_1 = m_2 = m_3$ in (1) in order to get the same vector mesons masses in the chiral limit. The mass splittings of the vector mesons from the quark mass effect are reduced (17)–(21) although no explicit vector meson mass terms generated from the leading order quark mass expansion appear in the effective lagrangian. Due to the definition of the physical vector mesons, the $\rho^0 - \omega$ mixing of (14) should be translated into

$$\mathcal{L}_{\rho^0\omega} = \frac{1}{4\pi^2 g^2}\frac{m_u - m_d}{m}\rho_{\mu\nu}^0 \omega^{\mu\nu}. \qquad (22)$$

The higher order terms in quark mass expansion have been ignored in the above equation.

3 $\rho^0 - \omega$ mixing amplitude and quark mass parameters

The $\rho^0 - \omega$ mixing induced by the isospin symmetry breaking has been derived in Sec. 2 (22). Now, we try to get the contribution of $\rho^0 - \omega$ mixing from electromagnetic interactions. The photon field has been introduced in (1). Following [17,18], the direct couplings of neutral vector meson fields (ρ^0, ω, and ϕ) and the photon fields read

$$\mathcal{L}_{\rho\gamma} = -\frac{1}{2}\frac{e}{f_\rho}\rho_{\mu\nu}^0(\partial^\mu A^\nu - \partial^\nu A^\mu), \qquad (23)$$

$$\mathcal{L}_{\omega\gamma} = -\frac{1}{2}\frac{e}{f_\omega}\omega_{\mu\nu}(\partial^\mu A^\nu - \partial^\nu A^\mu), \qquad (24)$$

$$\mathcal{L}_{\phi\gamma} = -\frac{1}{2}\frac{e}{f_\phi}\phi_{\mu\nu}(\partial^\mu A^\nu - \partial^\nu A^\mu), \qquad (25)$$

where

$$\frac{1}{f_\rho} = \frac{1}{2}g, \quad \frac{1}{f_\omega} = \frac{1}{6}g, \quad \frac{1}{f_\phi} = -\frac{1}{3\sqrt{2}}g. \qquad (26)$$

Eqs. (23) and (24) will lead to $\rho^0 - \omega$ mixing at the order of α_{EM} through the transition process $\rho \to \gamma \to \omega$, which is

$$\mathcal{L}_{\rho\omega} = \frac{1}{24}e^2 g^2 \rho_{\mu\nu}^0 \omega^{\mu\nu}. \qquad (27)$$

From (22) and (27), the total $\rho^0 - \omega$ mixing is

$$\mathcal{L}_{\rho\omega} = \frac{1}{4\pi^2 g^2} \frac{m_u - m_d}{m} \rho^0_{\mu\nu} \omega^{\mu\nu} + \frac{1}{24} e^2 g^2 \rho^0_{\mu\nu} \omega^{\mu\nu}. \quad (28)$$

Note that the vector mesons mass terms do not lead to any $\rho^0 - \omega$ mixing, therefore, the mixing $\mathcal{L}_{\rho\omega}$ in the present theory comes entirely from the quark loop. In the standard way, the two-point Green function associated with $\rho^0 - \omega$ mixing is

$$\Pi^{\rho\omega}_{\mu\nu}(q^2) = i \int d^4x \, e^{iqx}$$
$$\times \langle 0 | T \rho_\mu(x) \omega_\nu(0) \exp\{i \int d^4y \mathcal{L}_{\rho\omega}(y)\} | 0 \rangle,$$
$$= (g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) \frac{\theta^{\rho\omega}(q^2)}{(q^2 - m_\rho^2)(q^2 - m_\omega^2)}, \quad (29)$$

where

$$\theta^{\rho\omega}(q^2) = 2q^2 (\frac{m_u - m_d}{4\pi^2 g^2 m} + \frac{1}{24} e^2 g^2). \quad (30)$$

$\Pi^{\rho\omega}_{\mu\nu}(q^2)$ is of transversal structure, which is due to the the conservation of the vector currents. From (30), the off-shell $\rho^0 - \omega$ mixing amplitude $\theta^{\rho\omega}(q^2)$ is obviously momentum dependent, and vanishes at $q^2 = 0$. This is consistent with the argument by O'Connell et al. in [10] that this mixing amplitude must vanish at the transition from time-like to space-like four-momentum within a broad class of models.

Now, we consider the on-shell $\rho^0 - \omega$ mixing amplitude. From (30), we have

$$\theta^{\rho\omega}(m_\rho^2) = 2m_\rho^2 (\frac{m_u - m_d}{4\pi^2 g^2 m} + \frac{1}{24} e^2 g^2). \quad (31)$$

The mass difference of ρ and ω mesons has been ignored here. Note that the universal coupling constant g has been fixed in this theory [17–19,21], and m can be determined from eqs. (6), (7), and (8). Therefore, the electromagnetic contribution of the on-shell mixing amplitude [the second term in eq. (31)] is calculable. By taking $g=0.39$, $f_\pi=186$ MeV, and $m_\rho=768.5$ MeV, $\theta^{\rho\omega}(m_\rho^2)$ from the transition $\rho \to \gamma \to \omega$ is about 0.686×10^{-3} GeV^2. The $\rho^0 - \omega$ mixing leads to the G-parity forbidden decay of the ω meson, $\omega \to \rho^0 \to \pi^+\pi^-$. By using the experimental value of the decay width $\Gamma(\omega \to \pi^+\pi^-)$, one can get the total on-shell $\rho^0 - \omega$ mixing amplitude [5]. Therefore, by subtracting the electromagnetic contribution from the value extracted from this decay, the full contribution of the mixing amplitude due to isospin symmetry breaking will be obtained. This could provide an important constraint on the u-d quark mass difference.

Following [4,5,7], the decay width of the process $\omega \to \rho^0 \to \pi^+\pi^-$ is expressed as

$$\Gamma(\omega \to \pi^+\pi^-) = |\frac{\theta^{\rho\omega}(m_\rho^2)}{m_\omega^2 - m_\rho^2 - i(m_\omega \Gamma_\omega - m_\rho \Gamma_\rho)}|^2$$
$$\times \Gamma(\rho \to \pi^+\pi^-), \quad (32)$$

here Γ_ρ and Γ_ω are the widths of ρ and ω mesons respectively, and the decay width of $\rho \to \pi^+\pi^-$ has been calculated in [17]

$$\Gamma(\rho \to \pi^+\pi^-) = \frac{f_{\rho\pi\pi}^2}{48\pi} m_\rho (1 - \frac{4m_\pi^2}{m_\rho^2})^{\frac{3}{2}},$$
$$f_{\rho\pi\pi} = \frac{2}{g}\{1 + \frac{m_\rho^2}{2\pi^2 f_\pi^2}[(1 - \frac{2c}{g})^2 - 4\pi^2 c^2]\}. \quad (33)$$

Using the experimental data $B(\omega \to \pi^+\pi^-) = 2.21 \pm 0.30\%$ together with (32) and (33), we get

$$\theta^{\rho\omega}(m_\rho^2) = -(4.21 \pm 0.28) \times 10^{-3} \text{ GeV}^2, \quad (34)$$

which agrees well with the value $-(4.52 \pm 0.60) \times 10^{-3}$ GeV^2 obtained by Coon and Barrett [5]. The error bar in (34) is from the uncertainty in the branch ratio of the process $\omega \to \pi^+\pi^-$, and we have neglected the mass difference between m_ρ and m_ω and the width Γ_ω in the denominator of (32). The sign of the on-shell mixing amplitude has been discussed in [27], and it is determined from the relative phase of the ω to the ρ amplitude in the reaction $e^+e^- \to \pi^+\pi^-$ near m_ρ and m_ω.

More recently, the $\rho^0 - \omega$ mixing is investigated based on the analysis of $e^+e^- \to \pi^+\pi^-$ in [13,14]. Particularly, in [14], the magnitude, phase and the s dependence of the mixing have been determined from the pion form-factor in the timelike region, and the smaller absolute value of the on-shell mixing amplitude (compared with the value by Coon and Barrett [5]) has been obtained. As pointed out by O'Connell et al. in [12], the value of the mixing amplitude given in [5] is not the value which provides the optimal fit to the pion electromagnetic form-factor.

Combining (34) and (31), we can obtain the u-d quark mass difference

$$m_d - m_u = 6.14 \pm 0.36 \text{ MeV}. \quad (35)$$

The value of $m_u - m_d$ can also be extracted from the mass of the mesons. It has been known that, at the leading order in quark mass expansion, the mass difference of the non-strange mesons (π, a_1, and ρ) is almost entirely electromagnetic in origin, however, the mass difference of the strange mesons (K, K_1, and K^*) are from both electromagnetic interactions and isospin symmetry breaking effect. In [21], by employing $U(3)_L \times U(3)_R$ chiral theory of mesons, electromagnetic mass splittings of π, a_1, K, K_1, and K^* have been calculated to one loop order and $O(\alpha_{\text{EM}})$. In particular,

$$(m_{K^+}^2 - m_{K^0}^2)_{\text{EM}} = 0.002473 \text{ GeV}^2$$
$$= 2m_K \times 2.5 \text{ MeV}, \quad (36)$$

$$(m_{K^{*+}}^2 - m_{K^{*0}}^2)_{\text{EM}} = -0.003147 \text{ GeV}^2$$
$$= -2m_{K^*} \times 1.76 \text{ MeV}. \quad (37)$$

Using the experimental value of mass difference between K^+ and K^0 [28] together with (9)–(11), we can get

$$\frac{m_u + m_d}{m_u - m_d} = \frac{m_\pi^2}{(m_{K^+}^2 - m_{K^0}^2)_{\text{EXP}} - (m_{K^+}^2 - m_{K^0}^2)_{\text{EM}}}. \quad (38)$$

Then

$$m_u + m_d = 17.40 \pm 1.02 \text{ MeV}, \tag{39}$$

$$m_u = 5.64 \pm 0.32 \text{ MeV}, \quad m_d = 11.76 \pm 0.70 \text{ MeV}. \tag{40}$$

From (17), we get the vector meson mass in the chiral limit m_V is 759.5 ± 1.0 MeV. In fact, the value of $2\pi^2 g^2 m$ is about 745 MeV, which is not far from m_V. If we assume $m_V = 2\pi^2 g^2 m$, (17)–(21) will be simplified as follows

$$m_\rho = m_\omega = m_V + \frac{m_u + m_d}{2},$$

$$m_{K^{*\pm}} = m_V + \frac{m_u + m_s}{2},$$

$$m_{K^{*0}} = m_V + \frac{m_d + m_s}{2},$$

$$m_\phi = m_V + m_s,$$

which are different from the corresponding relations given by Urech [7],

$$m_\rho = m_\omega = m_V + 2\hat{m},$$

$$m_{K^{*\pm}} = m_{K^{*0}} = m_V + \hat{m} + m_s,$$

$$m_\phi = m_V + 2m_s,$$

where $\hat{m} = \frac{m_u + m_d}{2}$.

Taking $m_\phi = 1019$ MeV in eq. (21), the value of m_s is about 254 MeV. Thus, from (19) and (20), we can predict

$$m_{K^{*\pm}} = 892.13 \text{ MeV}, \quad m_{K^{*0}} = 895.27 \text{ MeV}. \tag{41}$$

The experimental data from [28] are

$$m_{K^{*\pm}} = 891.59 \pm 0.24 \text{ MeV},$$

$$m_{K^{*0}} = 896.10 \pm 0.28 \text{ MeV}. \tag{42}$$

The small differences between (41) and (42) are due to the electromagnetic corrections of the K^* mesons. From (41) and (37) [the value of $(m_{K^{*\pm}} - m_{K^{*0}})_{\text{EM}}$], the total mass difference of K^*-mesons is 4.90 MeV, which is in good agreement with the data of (42) ($m_{K^{*0}} - m_{K^{*\pm}})_{\text{exp}}$=4.51±0.52 MeV.

4 Summary

In order to calculate the $\rho^0 - \omega$ mixing amplitude, $U(3)_L \times U(3)_R$ chiral theory of mesons is extended to the leading order in quark mass expansion. The use of path integral method to integrate out the quark fields naturally leads to the interactions of the $\rho - \omega$ mixing. It has been shown that there is no explicit vector mass-mixing term in the effective lagrangian of this theory, and the $\rho^0 - \omega$ mixing comes entirely from the quark loop. The off-shell mixing amplitude is momentum dependent, and vanishes at $q^2 = 0$. The on-shell mixing amplitude is obtained from the G-parity forbidden decay $\omega \to \pi^+ \pi^-$, and the result is in agreement with the generally accepted value. The current quark masses are extracted from the on-shell

$\rho^0 - \omega$ mixing amplitude and the mesons masses. In particular, the masses of charge and neutral K^*-mesons are predicted, and their total mass difference from both the electromagnetic and isospin symmetry breaking effects are obtained.

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